

6.5

The Power of Interest (It's a Curious Thing)

Geometric Series Applications

LEARNING GOALS

In this lesson you will:

- Apply your understanding of series to problem situations.
- Write the formula for a geometric series representing a problem situation.

Imagine walking up to the counter of a major electronics store to purchase a new flat screen television for \$999. When the salesperson rings up your purchase, she says: "After fees and interest charges, your total is \$1950!" Would you still buy the television?

In another scenario, imagine walking into the tuition office at a local university. You are interested in taking 2 classes this fall while working part-time. The cost of the 6 credits is supposed to be \$5000, but after discussing a particular payment option with the financial aid officer, you realize that the classes will cost more than \$10,000 over the next 12 years. Would you still accept that payment option?

It may surprise you, but many people accept these terms for their purchases every day and don't even realize it. You may wonder how this could happen. The answer lies in the mathematics behind credit cards. When used wisely, credit cards can be convenient and flexible. They allow consumers to purchase expensive or necessary items and pay for them at a later date. On the other hand, if used unwisely, consumers waste a lot of money on interest charges.

How do credit cards work? How could you end up paying twice the amount for a television, or make monthly payments and still carry a balance on two college classes that you took over a decade ago?

PROBLEM 1 I Don't Want Credit For This

Vince wants to purchase a laptop with high screen resolution for his gaming hobby. He charges the \$1000 purchase to a credit card with 19% interest.

The credit card company requires a minimum monthly payment of the greater of:

- 2% of the balance on the card, or
- \$15.00.

To determine how long it will take to pay off the credit card when paying the minimum balance, Vince calls the company. He learns that when making a monthly payment, 75% of the minimum payment goes toward interest and the remaining portion of the minimum monthly payment goes toward the principal.

1. Determine the percent of the payment that is paid toward interest and principal for each monthly payment.
 - a. Monthly payment is 2% of balance.
 - b. Monthly payment is 10% of balance.
 - c. Monthly payment is 25% of balance.

Calculating finance charges is a very complex endeavor. The calculations in this lesson closely approximate what happens in real life.



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Let's consider Vince's monthly payment.

In order to calculate the minimum monthly payment, you will need to calculate 2% of the balance.

$$\begin{aligned}\text{minimum payment} &= (0.02)(\text{balance}) \\ &= 0.02(1000) \\ &= 20\end{aligned}$$

If the credit card balance is \$1000.00, the minimum monthly payment would be \$20.00.

The amount paid toward interest is 1.5% of the balance.

$$\begin{aligned}\text{amount paid toward interest} &= 0.015(\text{balance}) \\ &= (0.015)(1000) \\ &= 15\end{aligned}$$

If the minimum monthly payment is \$20.00, the amount paid toward interest would be \$15.00.

The amount paid toward principal is 0.5% of the balance.

$$\begin{aligned}\text{amount paid toward principal} &= (0.005)(\text{balance}) \\ &= (0.005)(1000) \\ &= 5\end{aligned}$$

If the minimum monthly payment is \$20.00, the amount paid toward principal would be \$5.00.

The remaining balance on the credit card will be the current balance minus the amount paid toward principal.

$$\begin{aligned}\text{remaining balance} &= (\text{current balance}) - (\text{amount paid toward principal}) \\ &= 1000 - 5 \\ &= 995\end{aligned}$$

So, if Vince makes a monthly payment of \$20.00 on the \$1000.00 balance, the balance after the monthly payment is \$995.00.

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2. Calculate the monthly payment details for the first 12 months of minimum payments. The first row of the table reflects the calculations from the worked example.

Number of Months (n)	Balance Before Monthly Payment (\$)	Minimum Monthly Payment (\$)	Amount Paid Toward Principal (\$)	Amount Paid Toward Interest (\$)	Balance After Monthly Payment (\$)
1	1000.00	20.00	5.00	15.00	995.00
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
n					

3. Consider the worked example and the answers to each part to complete the n -row of the table.
- Write the explicit formula for the geometric sequence represented in the "Balance Before Monthly Payment" column.
 - Write the formula to calculate the minimum monthly payment.

To write the explicit formulas of each column in the n -row, consider the initial value and the rate of change.



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c. Write the formula to calculate the amount paid toward interest.

d. Write the formula to calculate the amount paid toward principal.



e. Write the formula to calculate the balance after minimum payment.

Vince knows how much he is paying each month, but it would be helpful if he knew how much he had paid in both interest and principal over a certain amount of time instead of on any given month.



4. Calculate each.

a. the total monthly payment in the first 12 months

You developed two different formulas to compute a geometric series. Which one are you going to use in this situation?

b. the amount paid toward principal in the first 12 months



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c. the amount paid toward interest in the first 12 months

5. Write a formula for the geometric series that represents:

- a. the total monthly payment.
- b. the total payment toward principal over time.
- c. the total payment toward interest over time.



6. Use the formulas you created in Question 5 to complete the table.

	Total Amount Paid Toward Principal	Total Amount Paid Toward Interest
2 years		
5 years		



7. Assume the credit card company does not require a monthly payment of \$15.00. Determine how long will it take to pay off the credit card completely.

8. Does your answer to Question 7 seem reasonable? Explain your reasoning.

Use a graphing calculator to help solve these problems.



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9. Determine the amount of time it will take to pay off the credit card balance, taking into account the minimum payment of \$15.00.
- After how many months will the minimum monthly payment become \$15.00?
 - What will be the balance on Vince's credit card when the minimum monthly payment becomes \$15.00?
 - How much will Vince pay toward principal with every \$15.00 payment?
 - How much will Vince pay toward interest with every \$15.00 payment?
 - How many months will Vince have to make the \$15.00 minimum payment to pay off the entire balance?
 - How many years will it take Vince to pay off the entire balance?

10. How much money will Vince end up spending on his \$1000.00 laptop?

Keep in mind,
Vince paid 2% of the
balance for awhile, and then
he made \$15 monthly payments
until the balance was paid
in full.

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11. How much money will Vince spend in interest to pay for his \$1000.00 laptop?

PROBLEM 2 A Little Less Interest-ing



After realizing how long it would take to pay his entire credit card bill when only paying the minimum amount, Vince decides that he should pay more than the minimum amount every month. Vince determines that he can pay 10% of the balance every month.

Remember, amount paid toward interest is still 75% of the monthly payment. You might want to use a spreadsheet to perform these calculations.



1. Complete the table to represent this information for 12 months.

Number of Months (n)	Balance Before Monthly Payment (\$)	10% Monthly Payment (\$)	Amount Paid Toward Principal (\$)	Amount Paid Toward Interest (\$)	Balance After Monthly Payment (\$)
1	1000.00	$0.10(1000) = 100$	25.00	75.00	975.00
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
n					

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2. Write the formula for the geometric series that represents:

- a. the total monthly payment.
- b. the total payment toward principal over time.
- c. the total payment toward interest over time.

Use the formulas from the table in Problem 1 to help you with the formulas for this table.



3. Use the formulas you created in Question 6 to complete the table.

	Total Money Spent in Principal	Total Money Spent in Interest
1.5 years		
2 years		

4. After how many months will the minimum monthly payment become \$15.00?

Remember the minimum payment is the greater of 2% of the balance or \$15.00.



5. What will the balance on his credit card, when the minimum payment becomes \$15.00?

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6. Vince decides that once he gets to the minimum payment, he will pay off the rest of the balance in one lump sum.
- How long will it take Vince to pay off the entire balance?
 - How much money will Vince end up spending on his \$1000.00 laptop?
 - How much money will Vince spend in interest to pay for his \$1000.00 laptop?
7. How much money will Vince save by paying 10% instead of the required 2% minimum payment?



PROBLEM 3 Enough of These Interest Payments

Vince is still concerned about paying so much money in interest on his credit card, and decides that he can afford to pay a flat amount of \$100 each month.



- Sean calculates how long it will take Vince to pay off his credit card this way.

 **Sean**

$$\frac{\$1000}{100} = 10$$

It will take Vince 10 months to pay off his credit card bill.

What is wrong with Sean's work? Explain your reasoning.

2. How long do you think it will take Vince to pay off his credit card this way?
Explain your reasoning.

3. Complete the table to represent this information for 12 months.

Number of Months (n)	Balance Before Monthly Payment (\$)	\$100 Monthly Payment (\$)	Amount Paid Toward Principal (\$)	Amount Paid Toward Interest (\$)	Balance After Monthly Payment (\$)
1	1000.00	100.00	25.00	75.00	975.00
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
n					

4. What pattern do you notice in this table that is different from the tables in Problems 1 and 2?

5. How long will it take Vince to pay off the entire balance?



6. How much money will Vince end up spending on his \$1000.00 laptop?



7. How much money will Vince spend in interest to pay for his \$1000.00 laptop?

PROBLEM 4 Interest Free? Whoopee!



Now that Vince has become more educated about credit card finances and proven that he can be responsible for paying off his debt, he applies for a new credit card that offers the first 6 months interest free for any purchases. Like his other credit card, this new credit card requires a minimum monthly payment of the greater of:

- 2% of the balance on the card, or
- \$15.00.



Vince is approved for the card, and charges \$1000 for a flat screen TV. He still decides to pay 10% of the balance after noticing how much money that saved him the last time.

How will your calculations change for month 7?



1. Complete the table to show 12 months of payments.

Number of Months (n)	Balance Before Monthly Payment (\$)	10% Monthly Payment (\$)	Amount Paid Toward Principal (\$)	Amount Paid Toward Interest (\$)	Balance After Monthly Payment (\$)
1	1000.00	$0.10(1000) = 100$	100.00	0.00	900.00
2					
3					
4					
5					
6					
Interest begins on the 7th month.					
7					
8					
9					
10					
11					
12					
n					

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2. Describe the change in Vince's balance in the first 6 months compared to the last 6 months.

3. Analyze the 12 months of payments. What recommendations would you give Vince about paying off his credit card? Include the number of months he would be paying on the balance, the total amount that he would pay for the flat screen TV, and the amount of total interest he would pay.



